[17] D. W. Kammler, "The Design of Discrete N-Section and Continuously Tapered Symmetrical TEM Directional Couplers," *IEEE Transactions on Microwave Theory and Techniques*, vol. MTT-17, pp. 577–590, August 1969.

PROBLEMS

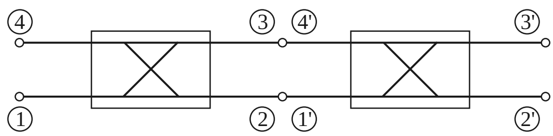
7.1 Write the scattering matrix for a nonideal symmetric hybrid coupler in terms of the coupler parameters, C, D, I, and L, as defined in (7.20). Repeat for a nonideal antisymmetric hybrid coupler. Assume that the couplers are matched at all ports.

7.2 A 20 dBm power source is connected to the input of a directional coupler having a coupling factor of 20 dB, a directivity of 35 dB, and an insertion loss of 0.5 dB. If all ports are matched, find the output powers (in dBm) at the through, coupled, and isolated ports.

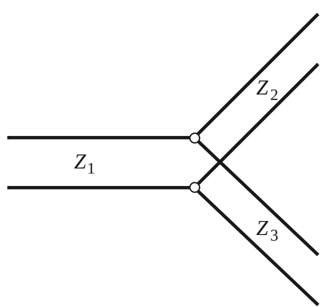
7.3 A directional coupler has the scattering matrix given below. Find the return loss, coupling factor, directivity, and insertion loss. Assume that the ports are terminated in matched loads.

$$[S] = \begin{bmatrix} 0.1\angle 40^\circ & 0.944\angle 90^\circ & 0.178\angle 180^\circ & 0.0056\angle 90^\circ \\ 0.944\angle 90^\circ & 0.1\angle 40^\circ & 0.0056\angle 90^\circ & 0.178\angle 180^\circ \\ 0.178\angle 180^\circ & 0.0056\angle 90^\circ & 0.1\angle 40^\circ & 0.944\angle 90^\circ \\ 0.0056\angle 90^\circ & 0.178\angle 180^\circ & 0.944\angle 90^\circ & 0.1\angle 40^\circ \end{bmatrix}$$

7.4 Two identical 90° couplers with $C = 8.34$ dB are connected as shown below. Find the resulting phase and amplitudes at ports $2'$ and $3'$, relative to port 1.

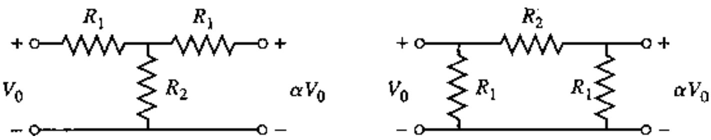


7.5 Consider the T-junction of three lines with characteristic impedances Z_1, Z_2, and Z_3, as shown below. Demonstrate that it is impossible for all three lines to be matched when looking toward the junction.

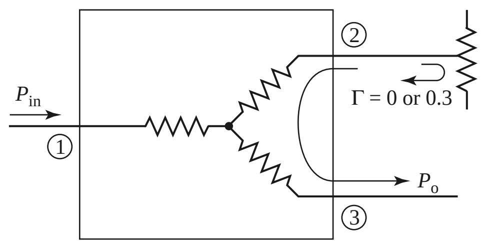


7.6 Design a lossless T-junction divider with a 30 Ω source impedance to give a 3:1 power split. Design quarter-wave matching transformers to convert the impedances of the output lines to 30 Ω. Determine the magnitude of the scattering parameters for this circuit, using a 30 Ω characteristic impedance.

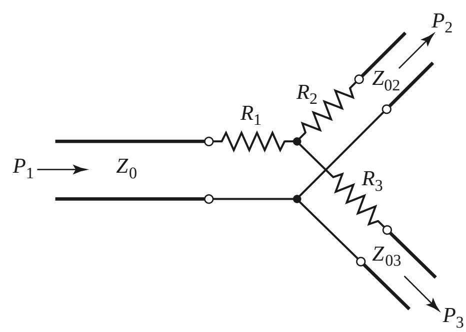
7.7 Consider the T and π resistive attenuator circuits shown below. If the input and output are matched to Z_0, and the ratio of output voltage to input voltage is α, derive the design equations for R_1 and R_2 for each circuit. If $Z_0 = 50$ Ω, compute R_1 and R_2 for 3, 10, and 20 dB attenuators of each type.



7.8 Design a three-port resistive divider for an equal power split and a 100 Ω system impedance. If port 3 is matched, calculate the change in output power at port 3 (in dB) when port 2 is connected first to a matched load, and then to a load having a mismatch of $\Gamma = 0.3$. See the figure below.



7.9 Consider the general resistive divider shown below. For an arbitrary power division ratio $\alpha = P_2/P_3$, derive expressions for the resistors R_1, R_2, and R_3, and the output characteristic impedances Z_{o2} and Z_{o3} so that all ports are matched, assuming the source impedance is Z_0.



7.10 Design a Wilkinson power divider with a power division ratio of $P_3/P_2 = 1/3$ and a source impedance of 50 Ω.

7.11 Derive the design equations in (7.37a)–(7.37c) for the unequal-split Wilkinson divider.

7.12 For the Bethe hole coupler of the type shown in Figure 7.16a, derive a design for s so that port 3 is the isolated port.

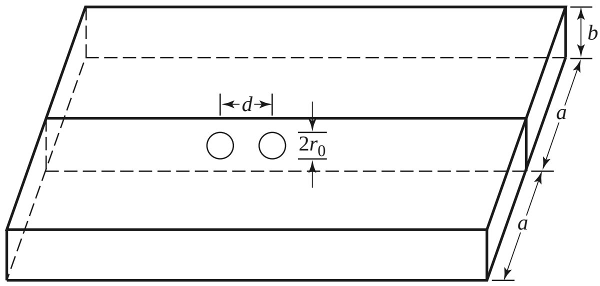
7.13 Design a Bethe hole coupler of the type shown in Figure 7.16a for a Ku-band waveguide operating at 11 GHz. The required coupling is 20 dB.

7.14 Design a Bethe hole coupler of the type shown in Figure 7.16b for a Ku-band waveguide operating at 17 GHz. The required coupling is 30 dB.

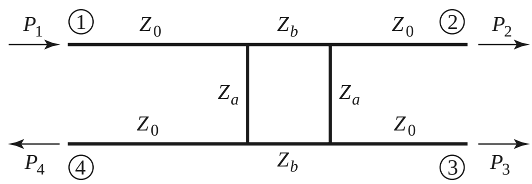
7.15 Design a five-hole directional coupler in a Ku-band waveguide with a binomial directivity response. The center frequency is 17.5 GHz, and the required coupling is 20 dB. Use round apertures centered across the broad wall of the waveguides.

7.16 Repeat Problem 7.14 for a design with a Chebyshev response, having a minimum directivity of 30 dB.

7.17 Develop the necessary equations required to design a two-hole directional coupler using two waveguides with apertures in a common sidewall, as shown below.



7.18 Consider the general branch-line coupler shown below, with shunt arm characteristic impedances Z_a and series arm characteristic impedances Z_b. Using an even-odd mode analysis, derive design equations for a quadrature hybrid coupler with an arbitrary power division ratio of $\alpha = P_2/P_3$, and with the input port (port 1) matched. Assume all arms are $\lambda/4$ long. Is port 4 isolated, in general?



7.19 An edge-coupled stripline with a ground plane spacing of 2.0 mm and a dielectric constant of 4.2 has strip widths of 0.6 mm and a separation of 0.2 mm between the edges of the strips. Use the graph of Figure 7.29 to find the resulting even- and odd-mode characteristic impedances. If possible, compare your results to those obtained from a microwave CAD tool.

7.20 A coupled microstrip line is to be designed for a substrate having a thickness of 2.0 mm and dielectric constant of 10.0. The required even- and odd-mode characteristic impedances are 133 Ω and 71.5 Ω, respectively. Use the graph of Figure 7.30 to find the required line widths and separation. If possible, compare your results to those obtained from a microwave CAD tool.

7.21 Repeat the derivation in Section 7.6 for the design equations of a single-section coupled line coupler using reflection and transmission coefficients instead of voltages and currents.

7.22 Design a single-section coupled line coupler with a coupling of 19.1 dB, a system impedance of 60 Ω, and a center frequency of 8 GHz. If the coupler is to be made in stripline (edge-coupled), with $\epsilon_r = 2.2$ and $b = 0.32$ cm, find the necessary strip widths and separation.

7.23 Repeat Problem 7.22 for a coupling factor of 5 dB. Is this a practical design?

7.24 Derive Equations (7.83) and (7.84).

7.25 A 20-dB three-section coupled line coupler is required to have a maximally flat coupling response with a center frequency of 3 GHz and $Z_0 = 50$ Ω. (a) Design the coupler and find Z_{0e} and Z_{0o} for each section. Use CAD to plot the resulting coupling (in dB) from 1 to 5 GHz. (b) Lay out the microstrip implementation of the coupler on an FR4 substrate having $\epsilon_r = 4.2$, $d = 0.158$ cm, and $\tan\delta = 0.02$, with copper conductors 0.5 mil thick. Use CAD to plot the insertion loss versus frequency.

7.26 Repeat Problem 7.25 for a coupler with an equal-ripple coupling response, where the ripple in the coupling is 1 dB over the passband.

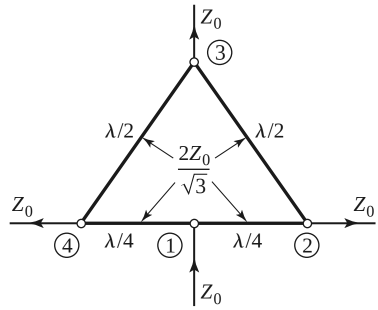
7.27 For the Lange coupler, derive the design equations (7.100) for Z_{0e} and Z_{0o} from (7.98) and (7.99).

7.28 Design a 3 dB Lange coupler for operation at 5 GHz. If the coupler is to be fabricated in microstrip on an alumina substrate with $\epsilon_r = 10$ and $d = 1.0$ mm, compute Z_{0e} and Z_{0o} for the two adjacent lines, and find the necessary spacing and widths of the lines.

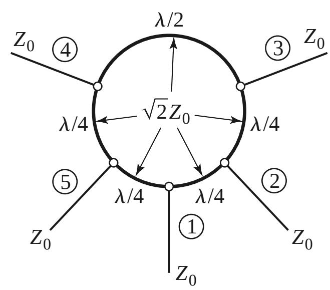
7.29 An input signal V_1 is applied to the sum port of a 180° hybrid, and another signal V_4 is applied to the difference port. What are the output signals?

7.30 Calculate the even- and odd-mode characteristic impedances for a tapered coupled line 180° hybrid coupler with a 3 dB coupling ratio and a 50 Ω characteristic impedance.

7.31 Find the scattering parameters for the four-port Bagley polygon power divider shown below.



7.32 For the symmetric hybrid shown below, calculate the output voltages if port 1 is fed with an incident wave of $1\angle 0$ V. Assume that the outputs are matched.



7.33 The Bailey unequal-split power divider uses a 90° hybrid coupler and a T-junction, as shown below. The power division ratio is controlled by adjusting the feed position, a, along the transmission line of length b that connects ports 1 and 4 of the hybrid. A quarter-wave transformer of impedance $Z_0/\sqrt{2}$ is used to match the input of the divider. (a) For $b = \lambda/4$, show that the output power division ratio is given by $P_3/P_2 = \tan^2(\pi a/2b)$. (b) Using a branch-line hybrid with $Z_0 = 50\ \Omega$, design a power divider with a division ratio of $P_3/P_2 = 0.5$, and plot the resulting input return loss and transmission coefficients versus frequency.